The Application of Process Capability Index in Improving Service Quality

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***Abstract*—the paper [1] use error process capability index C_{pk} to improve the quality of service, this paper use modified process capability index C_{pkr}, compared with C_{pkr} ,The specific meaning of every index of C_{pk} and C_{pkr} was analyzed quantitatively through the case of the insurance industry and points out the calculated value of error in paper .**

***Index Terms*—Process capability index; Prepare process capability index; Process capability index interval**

I. Introduction

The ability index that evaluates a process quality level to meet customer requirements is called process capability index(PCI)[]2-5].There are two aspects to this:process and the customer,extract information from different angles, and then compare them, and form a process capability index.Common ones include: C_p, C_{pk} , C_p. Indicates the process capability index of symmetry tolerance in the unbiased case,but In the production practice.The process output mean is not uncommon relative to the target value.Therefore, C_p is an idealized state, called the underlying process capability index, which cannot be used for the calculation of the process ability index in the case of symmetric tolerance.Although the process capability index C_{pk} considers the deviation of the process output mean relative to the target value,but there are serious shortcomings in practical application.For example, when a is zero, the acceptance rate is still about 50%. Although C_{pk} is used today, and C_{pk} seems to be highly receptive,but the debate over C_{pk} has not stopped since its birth.The error of a is that the calculated process capacity is always on the low side. The process ability index is zero when the mean is shifted to the upper and lower bounds of tolerance.The process capability index, which moves out of tolerance, becomes negative.The formula is still used by experts and scholars around the world, since there has been no one to correct C_{pk}'s error and put forward the correct formula.It is Mr Song who has made outstanding contributions to this.Using mathematical tools and methods, he discovered four inferences and four essential properties of the process capability index, based on the definition of process capability index C_p [6-8]. At the same time, the error of C_{pk} is proved by various methods, and the formula C_{pkr} of the revised process ability index is obtained[9-10].

II. Know C_p

The process capability index C_p and C_{pk} are carried out in the following three assumptions: 1, Process stability (controlled), namely the fluctuation of the quality of the process only caused by normal fluctuation, so the distribution of process output does not change over time. The future state of the process is predictable; 2, The Quality characteristics X of the process is subject to a normal distribution $N(\mu,\sigma^2)$; 3, LSL and USL, the bilateral specification limit, can accurately express customer requirement.

The process capability index involves the process and the customer. When the process stability and quality characteristics X Obey the normal distribution $N(\mu,\sigma^2)$. The product's quality attribute value x of 99.73 percent of the range $[\mu-3\sigma,\mu+3\sigma]$ has length 6σ, (as shown in figure 1), which is the process capability of the process(PC),notes for $PC=6\sigma$.

Customer requirements are shown in the specification limit (LSL,USL), the point $M=(LSL+USL)/2$ is called the canonical center. The width $T=USL-LSL$ of the specification limit is often called the tolerance, it represents the loose and severity of the customer's request.

When the canonical center M and the controlled process center (namely the normal mean μ) overlap, the process capability index is defined as

$$C_p = \frac{Customer\ requirements}{Process\ capability} = \frac{USL-LSL}{6\sigma} = \frac{T}{6\sigma}$$

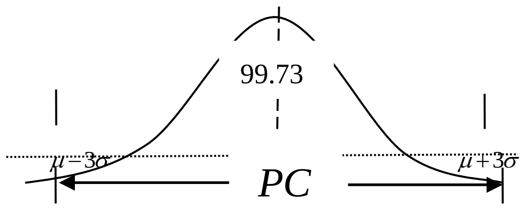

Figure 1 Process capability

In this definition, the specification limit (LSL, USL) is a customer requirement that is generally not easily change, so C_p is inversely proportional to σ, σ is as small as possible, C_p is a bigger and better index.

III. THE ANATOMY OF THE FORMULA FOR C_{pk}

Improve C_p to meet the actual production process, the key is to bring the process center μ to the index. Normally, process center μ is in the specification limits (LSL, USL), and the specification limits is divided into two subdivisions: (LSL, μ) and (μ, USL).The ratio of them and 3σ reflects the extent to which the process meets the customer's requirements on the left or right(as shown in figure 2).We call $C_{pL} = \frac{\mu - LSL}{3\sigma}$ The single-lateral lower limit process capability index $C_{pU} = \frac{USL-\mu}{3\sigma}$. The single-lateral upper limit process capability index

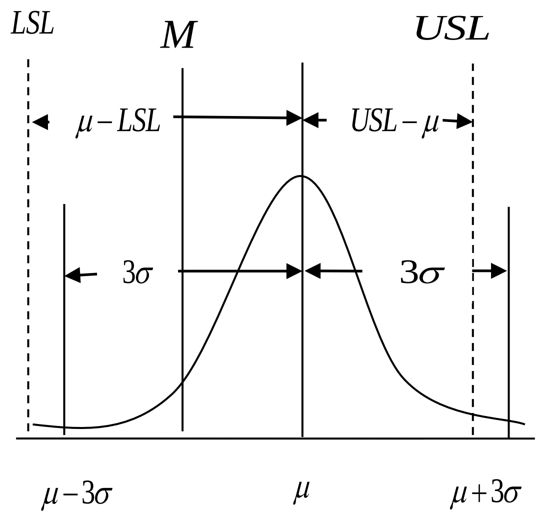

Figure 2 The mean and the tolerance don't overlap

When $M \neq \mu$, $C_{pL} \neq C_{pU}$, This is because $\mu - LSL \neq USL - \mu$.In terms of improving quality, we should focus on the smaller ones in C_{pL} and C_{pU},we call

$C_{pk} = \min\{C_{pL}, C_{pU}\}$ the actual process capability index.

$C_p = \frac{USL-LSL}{6\sigma} = \frac{T}{6\sigma}$ the underlying process capability index.

We can see from the definition of C_{pk}:

$$C_{pk} = \frac{\min\{USL-\mu, \mu - LSL\}}{3\sigma}.$$

Using the identity

$$\min\{a,b\} = \frac{1}{2}(|a+b| - |a-b|).$$

then the numerator of C_{pk} Can be rewrite into

$$\frac{1}{2}(USL - LSL - |USL + LSL - 2\mu|)$$

using $T = USL - LSL, M = (USL + LSL)/2$,we can get

$$C_{pk} = \frac{T}{6\sigma} - \frac{|M-\mu|}{3\sigma} = \frac{T}{6\sigma} - \frac{2T|M-\mu|}{T \bullet 6\sigma}$$

$$= \frac{T}{6\sigma}(1 - \frac{2|M-\mu|}{T}) = C_P(1-K).$$

Among them $K = \frac{2|M-\mu|}{T}$, let $\varepsilon = |M-\mu|$, then

$$C_{pk} = \frac{T}{6\sigma} - \frac{\varepsilon}{3\sigma}$$

The previous term $\frac{T}{6\sigma}$ is the process capability index of the unbiased case, the previous term $\frac{\varepsilon}{3\sigma}$ is process capability index that is lost due to offsets. We know that,the definition of the process capability index is the amount of technical tolerance T divided by 6σ,and here, 6σ is having its particular meaning, it represents the process capacity, but 3σ doesn't mean process capability, therefore, $\frac{\varepsilon}{3\sigma}$ does not represent the process capability index, so $\frac{\varepsilon}{3\sigma}$ can't represent the process capability index that is lost because of offsets, $\frac{T}{6\sigma} - \frac{\varepsilon}{3\sigma}$ cannot represent the process capability index after offsets.

Formula through the above analysis, we realize that C_{pk} is a flawed formula, we propose a modified process capability index formula C_{pkr} below.

Ⅳ. PROPOSE A MODIFIED FORMULA C_{pkr}

When the normal distribution curve is shifted to the right by $\frac{T}{2}$, according to the process capability index basic feature 4,the contribution of the percent of pass within the range of $[T_L, M]$ to the process capability index is zero,and the percent of pass of the range of $[M, T_U]$ is half of the normal distribution curve In the absence of deviation,at this time, the contribution of this half to the process capability index is $C_p/2$,after the migration, this half of the rate is still in the qualified area, and its contribution to the process capability index does not change due to deviation, it is still $C_p/2$.

Because $\Delta = 0$, and because the deviation process ability index formula is a linear equation for the independent variable ε ,and this line goes through two points $(0, C_p)$ and $(T/2, C_p/2)$,assuming that $C_{pkr} = a\varepsilon + b$, so let's plug these two into the equation,we can get:

$$\begin{cases} C_p = 0 + b \\ \frac{1}{2}C_p = a \bullet \frac{T}{2} + b \end{cases} \Rightarrow \begin{cases} a = -\frac{C_p}{T} \\ b = C_p \end{cases}$$

So

$$C_{pkr} = -\frac{C_p}{T}\varepsilon + C_p = (1 - \frac{\varepsilon}{T})C_p = (1 - \frac{1}{2}\frac{2\varepsilon}{T})C_p$$

$$C_{pkr} = -\frac{C_p}{T}\varepsilon + C_p = (1 - \frac{\varepsilon}{T})C_p$$

$$= (1 - \frac{1}{2}\frac{2\varepsilon}{T})C_p = (1 - \frac{1}{2}K)C_p .$$

Ⅴ. THE COMPARISON OF C_{pk} AND C_{pkr}

Let's say that the ε is the X-axis, the dependent variable PCI is the vertical axis, because the process ability index formula is a simple equation of ε, and the correct C_{pkr} goes through two points $(0, C_p)$ and $(T/2, C_p/2)$, so the equation of C_{pkr} is determined by these two points, (as shown in figure 3), and the wrong equation of C_{pk} is

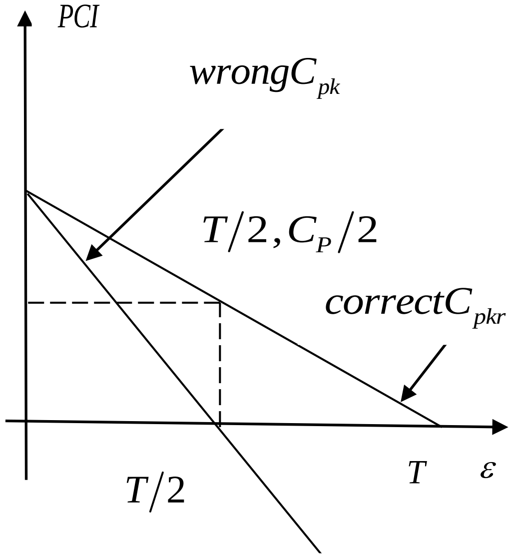

Figure 3 The comparison of C_{pkr} and C_{pk}

going to be through $(0, C_p)$ and $(T/2, 0)$, $(0, C_p)$ is correct, and the physical significance of b is wrong, it violates the symmetry of the normal distribution, so the equation of C_{pk} is also wrong, and they have the following relationship: $C_{pkr} - C_{pk} = \frac{\varepsilon}{6\sigma}$.

Ⅵ. THE EMPIRICAL ANALYSIS

It usually takes at least five days for an insurance company to make claims on a certain type of event, the company promises to deal with all claims for up to 12 days.This is the summary of the company for the past experience, think at the same time, the general average processing time is 9 days, this is to reflect the average proficiency in handling business, in the past month, the company's departments A dealt with 50 pieces of claim event, the time required is as follows(0. 5 for half a day)

11. 5, 10. 5, 9. 0, 9. 5, 10. 5, 9. 5, 9. 5, 11. 0,10. 0,9. 5, 11. 5, 7. 0, 9. 5, 10. 5, 7. 0, 9. 0, 10. 0,6. 5,9. 0,8. 0,8. 5,9. 5,9. 5,10. 0,11. 0,8. 5,8. 5,10. 0,9. 0,10. 0,10. 5,9. 0,11. 5,8. 5,9. 0,10. 0,11. 0, 8. 5, 8. 0, 10. 0, 9. 5, 9. 5, 10. 0, 9. 0, 9. 0,12. 0,8. 0,9. 5,12. 0,11. 5.

Then how should the department evaluate claims?

According to the correct formula C_{pkr} , $USL=12, LSL=5$, $M=(LSL+USL)/2=(12+5)/2=8.5$ .Using the excel, we can figure out the mean and variance of the sample data is 9.58 and 1.24837. Let's do it in terms of X and S . Mathematically they are the unbiased estimator of μ and σ . The offset $\varepsilon=|8.5-9.58|=1.08$, so $C_p=\frac{T}{6\sigma}= \frac{12-5}{6\times1.24837}=0.935$,thus $C_{pkr}=(1-\frac{\varepsilon}{T})C_p=(1-\frac{1.08}{7})\times0.935=0.79$ the calculated value was 0.65 with the wrong formula in the literature [1],by[1],we can get $C_{pkr}-C_{pk}=\frac{\varepsilon}{6\sigma}=\frac{1.08}{6\times1.24837}\approx0.14$ $=0.79-0.65$, If $C_{pkr}>1$,then you can think that the quality of service is normal,if 1. 33 < C_{pkr} <1. 67,that means the service meets the standards well,if $C_{pkr}<1$,that means that the quality of service is inadequate and needs to be improved,if C_{pkr} smaller,that suggests that the service is likely to be seriously out of control,because here $C_{pkr}=0.79<1$,so there is a shortage of service quality and needs to be improved.

That we should be how to improve? By the formula $C_{pkr}=-\frac{C_p}{T}\varepsilon+C_p=\frac{T}{6\sigma}-\frac{\varepsilon}{6\sigma}$, in order to improve C_{pkr} ,we can think about it in two ways:

On the one hand, reduce the ε, that is, move the curve to the center position $M=8.5$. If we reduce the average to 9.1, then C_{pkr} is 0.85, (in the literature [1], the wrong C_{pk} is 0.77), obviously, it has improved.

On the other hand, reduce the σ and minimize the volatility. In this case, on the basis of reducing the average, if we reduce the standard deviation σ to 0.9, then C_{pkr} is 1.18 , (In the literature [1], the wrong a is 1.07), obviously, it's up to the standard.

Let's take a look at the improvement of department A:

9. 0,10. 5,9. 0,9. 0,7. 5,9. 5,10. 0,8. 5,7. 0,9. 5,9. 5,9. 0,11. 5,9. 5,8. 5,10. 5,9. 0,9. 0,8. 5,8. 5,9. 5,9. 0,10. 5,10. 0,6. 5,8. 5,7. 5,9. 0,8. 5,9. 0,8. 0,9. 0,9. 5,9. 0,9. 5,9. 5,10. 0,8. 5,7. 0,8. 0,9. 0,9. 5,9. 5,9. 0,9. 0,8. 0,10. 0,9. 5,10. 0,8. 5.

According to the data, we can figure out the mean is 9.02, the standard deviation is 0.93, C_{pkr} is 1.16(the wrong C_{pk} in the literature [1] is 1.1), the standards have clearly been met.

Through the above analysis discussion, we can see that C_{pk} is equal to C_{pkr} only at one point, as long as there is deviation, there is $C_{pk}<C_{pkr}$ and $C_{pkr}-C_{pk}=\frac{\varepsilon}{6\sigma}$.

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REFERENCES

[1] J. H. Wu, J. R. Rong. “An effective way to improve service quality: process capability index method”. Science and Technology Management Research.2006,(6):207-208.

[2] X. S. Sun. “Review of process capability index ". Journal of the Shandong School of Industry and Commerce.2010(5):32-37.

[3] S. M. Sun. “Process capability index review”. Applied Probability Statistics.2004 (2):207-216

[4] Y. L. Han. “Basic process capability index analysis and application problem research”. Equipment Manufacturing Technique.2012(12):185-197

[5] Y. Cai. “Correctly recognize and use the process capability index C_p and C_{pk} " Mathematical Statistical Analysis and Application.2007(2):77-82.

[6] X. Y. Song. “nterpretation of the four basic features of the process capability index”. Standard Science.2010(1):33-39

[7] X. Y. Song. “Four basic features of the process capability index”. Standard Science.2009 (12):4-11.

[8] W. J. Huang. “Several opinions on the basic characteristics of the process capability index”. Standard Science. 2013 (1):79-82

[9] X. Y. Song. “Analyze the 12 erroneous views of formula C_{pk} ”. Standard Science.2010 (10):76-80.

[10] X. Y. Song. “Six sigma questioned”. Qingdao Press., 2005.